Technical Comments

Comment on "Numerical Solution of the Boundary-Layer Equations"

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E. KRAUSE[1] stated that the Smith and Clutter[2] method for integrating the boundary-layer equations requires "not only u and T, but also $\partial u/\partial x$ and $\partial T/\partial x$ for the start of the integration." That this statement is incorrect can be demonstrated from the transformed momentum equation (we consider only incompressible flow for simplicity):

$$f''' + \left(\frac{M+1}{2}\right)ff'' - M(f'^2 - 1) = x\left[f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right] \quad (1)$$

where $f' = df/d\eta = u/u_e$, $\eta = y(u_e/\nu x)^{1/2}$, and $M(x) = (x/u_e)(du_e/dx)$. Using a simple two-point finite-difference scheme for the x derivatives, Eq. (1) becomes

$$f''' + \left(\frac{M+1}{2}\right)ff'' - M(f'^2 - 1) = \left(\frac{x+\Delta x}{\Delta x}\right)[f'(f' - F') - f''(f - F)] \quad (2)$$

where $f = f(x + \Delta x, \eta)$ and $F = f(x,\eta)$.

Equation (2) can be solved for f, subject to the appropriate boundary conditions, once F and F' are known. If at a particular value of x ($=x_0$), the initial velocity distribution u/u_e is known, then

$$F' = \frac{u}{u_e}(x_0,\eta) \qquad F = \int_0^\eta F'(x_0,\eta)d\eta \quad (3)$$

Equation (2) can then be used to determine the solution as one proceeds downstream using Eq. (3) as a starting condition. An analogous result applies to the compressible case.

References

[1] Krause, E., "Numerical solution of the boundary-layer equations," AIAA J. 2, 1231–1237 (1967).

[2] Smith, A. M. O. and Clutter, D. W., "Solution of the incompressible laminar boundary-layer equations," AIAA J. 1, 2062–2071 (1963).

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IF only the tangential velocity component and temperature are prescribed at the initial station, a compatible normal velocity component can always be computed. This, for example, is demonstrated in Ref. 1 for the implicit finite-difference solution of Flügge-Lotz and Blottner.

The remark in Ref. 1 regarding Smith and Clutter's method is motivated by Eq. (9) of Ref. 2. That equation expresses the x-derivatives by means of a three-point difference formula, presumably to achieve high accuracy. When the three-point formula is used to start the integration, it is necessary to specify the initial data at two previous stations instead of one. This is equivalent to specifying the function and its x-derivative at one station. The suggestion of the comment to use a two-point difference formula for the x-derivative does, in principle, eliminate the difficulties in starting the integration.

References

[1] Krause, E., "Numerical solution of the boundary-layer equations," AIAA J. 2, 1231–1237 (1967).

[2] Smith, A. M. O. and Clutter, D. W., "Solution of the incompressible laminar boundary-layer equations," AIAA J. 1, 2062–2071 (1963).

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Comment on "A Source Model for Predicting the Drag Force on a Moving Arc Column"

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THE source model used by Otis[1] to describe an electric arc column in a crossflow, is open to criticism 1) on the validity of applying this particular model to an arc and 2) on comparison with experimental results. Concerning 1, the following two main points may be made:

a) The relation $2\pi m = U_\alpha b_w$, (Sec. IIC), applies only to fluid that has the same density on both sides of the dividing streamline shown in Fig. 3 of Ref. 1. It is incorrect to assume that the preceding relation holds when the fluid emitted by the source is heated, so that the densities on the two sides of the dividing streamline are unequal.

b) The incompressible inviscid point source model adopted in Sec. IIC, since it is both a mass source and a volume source, contradicts the assumption implicit in the derivation of the mass and energy equations in Sec. IIA, which is that all fluid originates upstream of the zone of heating.

Apart from these two criticisms, other assumptions that might be disputed are the neglect of heat conduction and radiation in the energy equation of Sec. II.

Comparison of the source model with experimental results (criticism 2) is most conveniently made by considering the following equation obtained from the model, which is the form in which Otis also makes such a comparison:

$$c_\alpha B/E \simeq (k - 1)M_\alpha \quad (1)$$

where c_α is the freestream speed of sound, B the transverse magnetic field, E the column voltage gradient, k the ratio of

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